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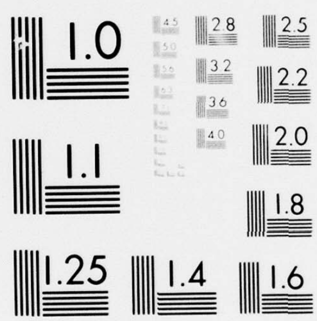

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Modified Kolomogorov-Smirnov Tests Which are Sensitive to Tail Behavior

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MODIFIED KOLOMOGOROV-SMIRNOV TESTS

WHICH ARE SENSITIVE TO TAIL BEHAVIOR[1]

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Several modified versions of 1-sample and 2-sample Kolmogorov-Smirnov tests are presented which have increased power against alternatives which differ from the hypothesized cdf (or cdf's) mainly in the tails. Asymptotic critical values are calculated. A Monte Carlo study gives an indication of the increase in power by examining one special case.

Key words: Kolmogorov-Smirnov statistics, Brownian motion, Brownian bridge

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1. Introduction.

For testing hypotheses $H_0:F=F_0$ (or $H_0:F=G$), the usual Kolmogorov-Smirnov statistics, $D_n=\sup|\hat{F}_n-F_0|$ (or $D_{n,n}=\sup|\hat{F}_n-\hat{G}_n|$), have very low power against alternatives which differ from F_0 mainly in the tails. To overcome this deficiency various weighted statistics have been suggested: such as $\sup w(F_0(t))|\hat{F}_n(t)-F_0(t)|$ and $\sup w((\hat{F}_n(t)+\hat{G}_n(t))/2)|\hat{F}_n(t)-\hat{G}_n(t)|$. Anderson and Darling(1952) use $w(x) = x(1-x)1_{[\delta,1-\delta]}(x)$, $\delta>0$. Dempster(1959), Dwass(1959) and others have looked at 1-sided test-statistics (generalized D^+-statistics) with $w(x) = (a+bx)^{-1}$. The purpose of this paper is to suggest a broad family of modified Kolmogorov-Smirnov statistics (which includes generalized D^+-statistics) which can be tailored to increase power against various alternative tail situations. These test-statistics retain the nice features of the Kolmogorov-Smirnov statistics such as consistency. We present the asymptotic (large sample) analysis of these test-statistics and a Monte Carlo study which will give some idea of the increase in power compared to the usual Kolmogorov-Smirnov statistics.

2. Modified Kolmogorov-Smirnov Statistics.

For the 1-sample tests define the stochastic process

$$K_n(t) = n^{1/2} (\hat{F}_n(F_0^{-1}(t)) - t), \quad 0\le t\le 1,$$

where F_0 is the hypothesized cdf and $\hat{F}_n$ is the empirical cdf based on n observations. For the 2-sample test (based on equal sample sizes) define the stochastic process

$$K_n(t) = (n/2)^{1/2} (\hat{F}_n(\hat{H}_n^{-1}(t))- \hat{G}_n(\hat{H}_n^{-1}(t))), \quad 0\le t\le 1,$$

where $\hat{H}_n = (\hat{F}_n+\hat{G}_n)/2$. We shall define <u>five different tests</u> of H_0 based on K_n. We define five subsets of $R^{[0,1]}$, the space of real-valued functions on $[0,1]$, which will be used as acceptance regions, see figure 1:

$$A_I(a,c) = \{ f : f(t) \le a(1+ct),\ 0\le t\le 1 \}$$

$$A_{II}(a,c) = \{ f : f(t) \le a(1+2c\cdot\min(t,1-t)),\ 0\le t\le 1 \}$$

$$A_{III}(a,c) = \{ f : |f(t)| \le a(1+ct),\ 0\le t\le 1 \}$$

$$A_{IV}(a,c) = \{ f : |f(t)| \le a(1+2c\cdot\min(t,1-t),\ 0\le t\le 1 \}$$

$$A_V(a,c) = \{ f : a(ct-(1+c)) \le f(t) \le a(1+ct),\ 0\le t\le 1 \}$$

To test H_0, we shall pick a region $A(a,c)$ and if $K_n(\cdot)$ falls in $A(a,c)$ accept the appropriate null hypothesis. The regions will be used as follows:

I. A_I will test $H_0{:}F\le F_0$ (or $H_0{:}F\le G$) and will be particularly sensitive to alternatives $F\ge F_0$, with $F>F_0$ only in the left tail. Analagously for the 2-sample case, $F\ge G$, with $F>G$ only in the left tail.

II. A_{II} will test $H_0{:}F\le F_0$ (or $H_0{:}F\le G$) but will be sensitive to tail-behavior in both tails.

III. A_{III} will test $H_0{:}F=F_0$ (or $H_0{:}F=G$) and will be particularly sensitive to alternatives where $F\neq F_0$ only in the left tail.

IV. A_{IV} will test $H_0{:}F=F_0$ (or $H_0{:}F=G$) and will be sensitive to alternatives where $F\neq F_0$ only in the right or left tail.

V. A_V will test $H_0{:}F=F_0$ (or $H_0{:}F=G$) and will be sensitive to alternatives where $F>F_0$ in the left tail or $F>F_0$ in the right tail, i.e. alternatives with heavier tails than F_0.

A rough qualitative explanation for the increased sensitivity of these regions to tail-behavior lies in the form of the test function, K_n. If H_0 is true then $EK_n(t)=0$ and for the 2-sample statistic K_n is a tied-down random walk, see Feller(1966), p. 70. If $F>G$ in the left tail then $EK_n(\delta)>0$ for small δ. An α-level region of the form A_I will detect this more readily in the presence of the random fluctuations than the usual α-level Kolmogorov-Smirnov acceptance region (which happens to be $A_I(a,0)$). This can be verified analytically or empirically. Increasing c increases the sensitivity to

tail behavior in the various acceptance regions (while changing a to keep the α-level). The precise shape of these regions was dictated by analytic tractibility.

3. Asymptotic Analysis: Critical Values.

If the underlying cfd's are continuous, the 1- and 2- sample test functions, K_n, converge weakly to the standard Brownian Bridge, $\{W^0(t),\ 0 \le t \le 1\}$, Billingsley(1968). The asymptotic acceptance probabilities

$$p_{A(a,c)} = \lim_{n\to\infty} P\{K_n \in A(a,c)\}$$

can be evaluated using the theory of weak convergence, namely:

$$p_{A(a,c)} = P\{W^0 \in A(a,c)\}.$$

These asymptotic acceptance probabilities are derived in the last section of the paper. We state the results here. Letting $p_{I,a,c} = P\{W^0 \in A(a,c)\}$, etc., these probabilities are:

$$p_{I,a,c} = 1 - \exp(-2a^2(c+1)), \quad a \ge 0,\ c \ge -1;$$

$$p_{II,a,c} = \Phi(2ac+2a) - 2\exp(-4a^2c-2a^2)\cdot\Phi(2ac) + \exp(-8a^2c)\cdot\Phi(2ac-2a),$$
$$a \ge 0,\ c \ge -1;$$

$$p_{III,a,c} = 1 + 2\sum_{k=1}^{\infty}(-1)^k\exp(-2k^2a^2(c+1)), \quad a \ge 0,\ c \ge -1;$$

$$p_{IV,a,c} = p_1\cdot p_2 - p_3\cdot p_4, \quad \text{where}$$
$$p_1 = \Phi(2a(c+1)) - \Phi(-2a(c+1)) + 2\sum_{n=1}^{\infty}\exp(-8a^2cn^2)(\Phi(2a(2n+c+1))-\Phi(2a(2n-c-1))),$$
$$p_2 = 1 + 2\sum_{n=1}^{\infty}\exp(-8a^2(c+1)n^2),$$
$$p_3 = 2\sum_{n=0}^{\infty}\exp(-2a^2c(2n+1)^2)(\Phi(2a(2n+1+c+1))-\Phi(2a(2n+1-c-1))),$$
$$p_4 = 2\sum_{n=0}^{\infty}\exp(-2a^2(c+1)(2n+1)^2),$$
$$a \ge 0,\ c \ge -1;$$

$$p_{V,a,c} = \sum_{k=-\infty}^{\infty}(\exp(-2ka^2(c+2)(k(c+2)-c))-\exp(-2a^2(1+k(c+2))(1+k(c+2)+c))),$$
$$a \ge 0,\ c \ge -1;$$

where Φ is the standard Normal cdf.

Asymptotic critical values of a for fixed α and c can be computed numerically from the above probabilities: $a_{I,c,\alpha}$ is the solution of $p_{I,a,c} = 1-\alpha$, etc. Selected critical values are tabulated in Table 1.

In closing this section we note that finite-sample-size analysis has been done for the two cases mentioned in section 1, see Dwass(1959) and Noe(1972). Noe has developed a general recursive method related to some work of Steck(1971) which gives numerical results in all cases. However, asymptotic results have apparently not been presented for all the regions listed in section 2.

4. A Monte Carlo Study of Power.

To get a rough idea of the sensitivity of these regions to tail behavior we performed the following Monte Carlo study. We tested H_0:F=G using A_V as the acceptance region, with $c = 0, 1, 2, 4,$ and 8, and $\alpha = .10$; we tested alternatives F, uniform(-1,1), against G, uniform(-s,s), s = 1, 1.1, 1.2, 1.3, and 1.4. The sample size was 40 observations from each. 1600 replicates were taken. The empirical power is tabulated in Table 2.

Several remarks are in order:

i) There is a dramatic increase in power for this case as c is increased.

ii) For sample size n=40 the asymptotic critical values are reasonable approximations for $0\leq c\leq 1$, but for greater values the convergence is much slower and for c=8 over one hundred observations are needed before it is a good approximation. However the error is always in the conservative direction.

iii) Note that c=0 corresponds to the usual Kolmogorov-Smirnov statistic and thus we have a comparison between the usual and modified statistics.

iv) Even these modified statistics are not very good at distinguishing alternatives such as Cauchy compared to Normal distributions. More specialized tests are required.

v) This Monte Carlo study is not intended to be complete or exhaustive. It is just an indication of the properties of these modified statistics and whether they might be of any value. In actual practice one would use the above work to make a rough selection of an acceptance region and then use the method of Noe(1971) to get precise probabilities for finite-sample size.

vi) For a related Monte Carlo study and analysis, see Canner(1975).

5. Theoretical derivation of asymptotic acceptance probabilities.

The asymptotic acceptance probabilities of section 3,

$$p_{A(a,c)} = \lim_{n\to\infty} P\{ K_n \in A(a,c)\}$$

are calculated using the theory of weak convergence of probability measures, Billingsley(1968), and properties of the Brownian motion W and the Brownian bridge W^0.

We begin with some preliminary properties of Brownian processes. Most of these are more-or-less well known. Those stated without proof can be found in (or derived from) Billingsley(1968). See also Breiman(1968) and Karlin and Taylor(1975) for treatments of Brownian motion. Lemma 9 is apparently new; it appears to be quite useful because it relates the usual Kolmogorov-Smirnov probabilities to those associated with $A_I(a,c)$ and $A_{III}(a,c)$ in a very simple way.

The standard Brownian motion $\{W(t),\ t\geq 0\}$ is a Gaussian process with 0 mean and covariance function equal to $\min(s,t)$. The standard Brownian bridge $\{W^0(t),\ 0\leq t\leq 1\}$ is Gaussian with 0 mean and covariance function

equal to min(s,t)-st. The sample paths of these processes lie in the space C[0,1]; see Billingsley (1968). A cylinder set is a set of the form $\{f \in C[0,1]: f(t_i) \leq a_i, i=1,\ldots,n\}$, for any $0 \leq t_1 < \ldots < t_n \leq 1$ and $a_1,\ldots,a_n$; B[0,1] is the smallest σ-field containing all cylinder sets; B[s,u] is the smallest σ-field containing all cylinder sets for which $s \leq t_1$ and $t_n \leq u$.

Lemma 1

If $A \in B[0,u]$, $u<1$, then $P\{W \in A | W(u)=a\} = P\{W^0 \in A | W^0(u)=a\}$ for almost all values of a.

Proof:

If A is a cylinder set, this is a statement about finite-dimensional distributions (f.d.d.'s) and follows from properties of the conditional distributions of multivariate normal distributions (Anderson(1958), p.27): it can be shown that both conditional probabilities are multivariate normal with mean vector $(t_i a/u)_i$ and covariance matrix $(t_i(1-t_j/u))_{i,j}$, $t_i \leq t_j$. The result then extends to any $A \in B[0,t]$ because the f.d.d.'s uniquely determine any probability measure on this σ-field.

Lemma 2 (Billingsley(1968), p.65)

Suppose $A \in B[0,1]$, then $P\{W-aI \in A | W(1)=a\} = P\{W^0 \in A\}$ for almost all a. (I is the identity function: I(t)=t.)

Lemma 3

The conditional distribution of W(t) given W(s)=a, W(u)=b, s<t<u, is Normal ([(u-t)a + (t-s)b]/(u-s), (t-s)(u-t)/(u-s)).

Lemma 4

$\{W^0(t), 0 \leq t \leq 1\}$ and $\{W^0(1-t), 0 \leq t \leq 1\}$ are identically distributed.

Lemma 5

$\{W(t)-W(s),\ s<t<1\}$ and $\{W(t-s),\ s<t<1\}$ are identically distributed.

Lemma 6 (Billingsley(1968), p. 72)

$$P\{\sup_{0\leq s\leq t} W(s)\leq x\} = \frac{2}{\sqrt{2\pi t}}\int_0^x \exp(-z^2/2t)dz = |N|(x/t^{\frac{1}{2}})$$

$x>0$, where $|N|$ is the cdf of the absolute value of a standard Normal random variable.

Lemma 7

$$P\{\sup_{0\leq s\leq t} W(s)\leq a \mid W(t)=x\} = 1 - \exp(-2(a^2-ax)/t) \text{ for almost all } x\leq a,\ \ a>0.$$

Proof

From a modification of equation 11.11 of Billingsley (1968), if $x \leq a$, then

$$P\{\sup_{0\leq s\leq t} W(s)\leq a,\ W(t)\leq x\} = P\{N_t<x\}-P\{2a-x<N_t\} \tag{5.1}$$

where N_t is a Normal(0,t) random variable. Letting ϕ_t be the density function of a Normal(0,t) r.v. and differentiating (5.1) with respect to x yields

$$P\{\sup_{0\leq s\leq t} W(s)\leq a|W(t)=x\} = \frac{\phi_t(x)-\phi_t(x-2a)}{\phi_t(x)},\ x<a$$

which is equivalent to the desired result.

Lemma 8

Let $m_t = \inf_{0\leq s\leq t} W(s)$ and $M_t = \sup_{0\leq s\leq t} W(s)$ then $P\{-a\leq m_t\leq M_t\leq b|W(t)=x\}$

$= [\phi_t(x)]^{-1} \sum_{k=-\infty}^{\infty} [\ \phi_t(x+2k(a+b))-\phi_t(x+2k(a+b)+2a)\ ]$, where ϕ_t is the density of a Normal(0,t) random variable.

Proof:

The proof is the same as Lemma 7 except that it follows from a modification of equation 11.10 of Billingsley(1968).

Lemma 9

Let g be a monotone increasing function, $g(0)=0$, $g(1)=1$; let f be any positive function. Then $\{f(t)W^0(t),\ 0 \le t \le 1\}$ and $\{W^0(g(t)),\ 0 \le t \le 1\}$ are identically distributed if and only if $f(t) = (1+c)^{1/2}/(1+ct)$ for some $c > -1$, in which case $g(t) = (c+1)t/(1+ct)$.

Proof:

Both processes are Gaussian with 0 mean. They will have the same distribution if and only if their covariance functions are the same, i.e.

$$f(s)f(t)(s(1-t)) = g(s)(1-g(t)) \ , \ s \le t \ . \tag{5.2}$$

Using separation of variables, it is easy to show that the only solution of (5.2) is the solution stated in the lemma.

From the probabilities which are calculated below, it will follow that the sets $A_{\cdot}(a,c)$ are continuity sets (Billingsley(1968), p.12) for W^0. For the 1-sample and 2-sample cases, K_n converges weakly to W^0; therefore

$$P_{A(a,c)} = \lim_{n\to\infty} P\{K_n \in A(a,c)\} = P\{W^0 \in A(a,c)\}$$

Thus we shall compute $P\{W^0 \in A(a,c)\}$ for the 5 different acceptance regions in section 2.

Theorem 1

$$P_{I,a,c} = P\{W^0 \in A_I(a,c)\} = P\{\sup_{0\le t\le 1} (1+ct)^{-1}W^0(t) \le a\} = 1 - \exp(-2(c+1)a^2).$$

Proof:

By Lemma 9, $(1+ct)^{-1}W^0(t)$ and $(c+1)^{-\frac{1}{2}}W^0((c+1)t/(ct+1))$ are identically distributed. Thus $P\{\sup_{0\le t\le 1}(1+ct)^{-1}W^0(t)\le a\}$ =

$P\{\sup_{0\le t\le 1}W^0(t)\le a(c+1)^{\frac{1}{2}}\} = 1-\exp(-2(c+1)a^2)$ from equation (11.40) of Billingsley.

Theorem 2

$$p_{III,a,c} = P\{\sup_{0\le t\le 1}|(1+ct)^{-1}W^0(t)|\le a\} = 1+2\sum_{k=1}^{\infty}(-1)^k e^{-2k^2(c+1)a^2}$$

Proof:

Follows from equation (11.39) of Billingsley and the above analysis.

Theorem 3

Let $f(t) = 1/(1+2ct)$, $0\le t\le 1/2$, and $= 1/(1+2c-2ct)$, $1/2\le t\le 1$.

$$p_{II,a,c} = P(\sup_{0\le t\le 1} f(t)W^0(t)\le a)$$

$$= \Phi(2ac+2a) - 2\exp(-4a^2c-2a^2)\Phi(2ac) + \exp(-8a^2c)\Phi(2ac-2a),\ a,c>0,$$

where $\Phi(x)$ is the standard Normal cdf.

Proof:

$$P(\sup_{0\le t\le 1} f(t)W^0(t)\le a)$$

$$= \int_{-\infty}^{a(1+c)} \left(P(W^0(t)\le a+2cat,\ 0\le t\le \tfrac{1}{2} \mid W^0(\tfrac{1}{2})=x)\right)^2 \phi_{\frac{1}{4}}(x)dx$$

By Lemma 9, the integrand equals

$$P\left(W^0(t)\le a\sqrt{2c+1},\ 0\le t\le \frac{2c+1}{2c+2} \;\Big|\; W^0\left(\frac{2c+1}{2c+2}\right) = \frac{x\sqrt{2c+1}}{c+1}\right)$$

$$= 1-\exp(-4(c+1)a^2+4ax)$$

using Lemmas 1 and 7. Combining the above expressions and carrying out the integration proves the theorem.

Theorem 4

Let $f(t) = 1/(1+2ct)$, $0 \le t \le 1/2$, and $= 1/(1+2c-2ct)$, $1/2 \le t \le 1$. Then

$$p_{IV,a,c} = P\{W^0 \in A_{IV}(a,c)\} = P(\sup_{0\le t\le 1} |f(t)W^0(t)| \le a) = C_1C_2 - C_3C_4 \quad \text{where}$$

$$C_1 = \Phi(2a(c+1))-\Phi(-2a(c+1))+2\sum_{n=1}^{\infty} \exp(-8a^2c(2n+1)^2)[\Phi(2a(2n+2+c))-\Phi(2a(2n-c))]$$

$$C_2 = 1 + 2\sum_{m=1}^{\infty} \exp(-8a^2(c+1)m^2)$$

$$C_3 = 2\sum_{n=0}^{\infty} \exp(-2a^2c(2n+1)^2)[\Phi(2a(2n+2+c))-\Phi(2a(2n-c))]$$

$$C_4 = 2\sum_{m=0}^{\infty} \exp(-2a^2(c+1)(2m+1)^2).$$

Proof:

The proof proceeds along the same line as the proof of Theorem 3 but uses Lemma 8 instead of 7 and thus becomes very tedious. We omit the calculations.

We conclude the paper by considering $P(f(t)\le W^0(t)\le g(t),\ 0\le t\le 1)$ where $f(t) = ct - (c+1)$ and $g(t) = ct + 1$. By Lemma 2, $P\{-(c+1)\le W\le 1 \mid W(1) = -a\} = P\{at-(c+1)\le W+at\le at+1,\ 0\le t\le 1 \mid W(1) = -a\} = P\{at-(c+1)\le W^0(t)\le at+1,\ 0\le t\le 1\}$, but this is only an almost everywhere statement. We don't know what version of the conditional probability to use when a=c. The following lemma will help.

Lemma 10

If $A = \{f: a\le f(t)\le b,\ 0\le t\le 1\}$, $a<0<b$, then a continuous version of $P(W \in A|W(1) = x)$ exists and equals $P\{a-xt\le W^0(t)\le b-xt,\ 0\le t\le 1\}$ everywhere.

Proof:

The continuous version of the conditional probability is given in Lemma 8. It remains to check that $P\{a-xt\le W^0(t)\le b-xt,\ 0\le t\le 1\}$ is continuous in x. Suppose $x_k \downarrow x$, then let $\Delta_k = \{f: a-xt\le f(t)\le b-xt\}$ $\Delta\{f: a-x_k t \le f(t) \le b-x_k t\}$; $\Delta_k \supset \Delta_{k+1}$; and $\cap_k \Delta_k = \{f: a-xt \le f(t) \le b-xt$ and $f(s) = b-xs$ for some $s\} \cup \{f: a-xt \le f(t) \le b-xt$ and $f(s) = a-xs$ for some $s\}$ $\subset \{f: \sup_t f(t)/(b-xt) = 1$ or $\inf_t f(t)/(a-xt) = 1\} = L$. By Theorem 1, $P\{W^0 \in L\} = 0$. Thus $\lim_{k\to\infty} P\{a-x_k t\le W^0(t)\le b-x_k t,\ 0\le t\le 1\} = P\{a-xt\le W^0(t)\le b-xt,\ 0\le t\le 1\}$. A similar result holds for $x_k \uparrow x$ and we have continuity. The two probabilities are equal a.s. and thus by continuity are equal everywhere.

Before proceeding we present a counterexample to the above lemma, namely, if $A = \{f \in C[0,1]: |f(t)| < (1-t)^{1/4}\}$ then $P(W \in A | W(1) = x) = 0$ almost surely but $P\{W^0 \in A\} > 0$. This follows from the law of the iterated logarithm, Breiman (1968), p. 263.

Theorem 6

Let $f(t) = a(ct-(c+1))$ and $g(t) = a(ct+1)$ then

$$\begin{aligned} P_{V,a,c} &= P\{f(t) \le W^0(t) \le g(t),\ 0 \le t \le 1\} \\ &= \sum_{k=-\infty}^{\infty} [\exp(-2k(c+2)[k(c+2)-c]a^2) - \exp(-2(1+k(c+2))(1+k(c+2)+c)a^2)] \end{aligned}$$

Proof:

By Lemma 5, this probability equals the continuous version of $P(-a(c+1)\le W(t)\le a,\ 0\le t\le 1 \mid W(1) = -ac)$ which is given in Lemma 8. The rest is substitution.

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<u>Figure 1</u>

Acceptance Regions

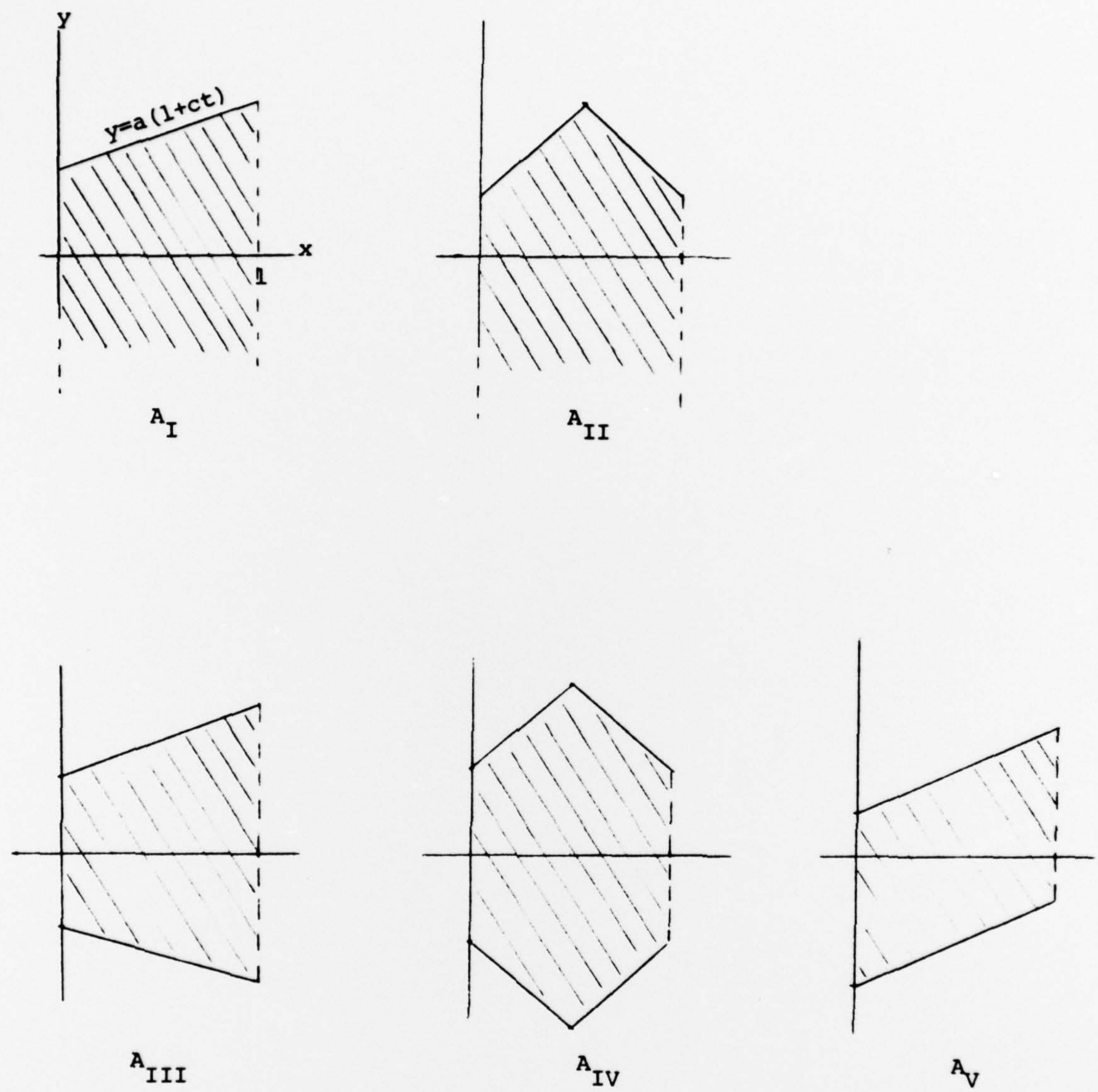

Table 1.

Critical values: $a_{\bullet,c,\alpha}$

Region	α-level	c = 0.0	0.5	1.0	2.0	4.0	8.0	16.0
A_I	.20	.8971	.7324	.6343	.5179	.4012	.2990	.2176
	.10	1.0729	.8760	.7587	.6195	.4798	.3577	.2602
	.05	1.2239	.9993	.8654	.7066	.5473	.4080	.2968
	.01	1.5174	1.2390	1.0730	.8761	.6786	.5058	.3680
A_{II}	.20	.8971	.6974	.5872	.4648	.3503	.2561	.1842
	.10	1.0729	.8213	.6862	.5391	.4044	.2950	.2120
	.05	1.2239	.9268	.7704	.6027	.4509	.3285	.2359
	.01	1.5174	1.1311	.9336	.7264	.5421	.3946	.2832
A_{III}	.20	1.0727	.8759	.7585	.6193	.4797	.3576	.2601
	.10	1.2238	.9993	.8654	.7066	.5473	.4080	.2968
	.05	1.3581	1.1089	.9603	.7841	.6074	.4527	.3294
	.01	1.6278	1.3290	1.1510	.9397	.7279	.5426	.3948
A_{IV}	.20	1.0727	.8208	.6855	.5382	.4033	.2939	.2111
	.10	1.2238	.9267	.7702	.6024	.4505	.3281	.2356
	.05	1.3581	1.0204	.8451	.6591	.4921	.3584	.2473
	.01	1.6277	1.2078	.9949	.7731	.5767	.4197	.3012
A_V	.20	1.0728	.8756	.7579	.6180	.4777	.3552	.2579
	.10	1.2238	.9992	.8652	.7062	.5466	.4071	.2959
	.05	1.3581	1.1089	.9603	.7840	.6071	.4524	.3290
	.01	1.6277	1.3288	1.1509	.9397	.7279	.5425	.3947

Table 2.

Power: testing Uniform(-1,1) vs. Uniform(-s,s)
acceptance region $A_V(\cdot,c)$
10% level of significance

		s				
		1.0	1.1	1.2	1.3	1.4
	0	.089	.101	.136	.197	.286
	1	.088	.118	.207	.349	.537
c	2	.069	.134	.271	.490	.664
	4	.059	.156	.349	.623	.806
	8	.049	.189	.490	.752	.906

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